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Trend Resistant Neighbour Balanced Bipartite Block Designs

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SUMMARY

This paper deals with bipartite block model with neighbour effects incorporating trend component. The information matrices for estimating direct as well as neighbour effects incorporating trend component have been derived. The conditions for a block design with neighbour effects to be trend resistant have also been obtained. Further, methods of constructing trend resistant neighbour balanced bipartite block design have been discussed. The designs so obtained are totally balanced for estimating direct and neighbour effects of treatments.

Keywords: Block design, Balanced bipartite, Neighbour effects, Direct effect, Trend.

1. INTRODUCTION

In designing of varietal trials, most of the times large scale experiments are required for identification of better varieties. Such situations sometimes require more than one control to check the performance of the new varieties. The experimental designs used for such situations are known as *bipartite designs*. Block designs are the most commonly used designs for many field experiments. In classical block design, it is assumed that the response from a unit/ plot to a particular treatment is not affected by the treatment applied on the neighbouring plots and the fertility associated with plots in a block is constant. However, in agricultural field experiments conducted in smaller units with no gaps, the estimates of treatment differences may deviate because of interference by the treatments applied in neighbouring units. For example, in an agricultural experiment, the response on a particular plot may be affected by the insecticide applied to that plot and also by the insecticide applied to its neighbouring plots. This interference or competition from neighboring units can contribute to variability in experimental results and lead to substantial loss in efficiency. Neighbour balanced designs may be useful for such situation. A lot of work is available in literature in this direction (Azais *et al*. 1993, Tomar *et al*. 2005, Jaggi *et al.* 2006, 2007). Abeyanayake *et al.* 2011 developed some series of neighbour balanced bipartite (NBB) designs for comparing a set of test treatments to a set of control treatments.

In some situations under block design set up, experiments may be carried out using plots occurring in long, narrow rows wherein fertility gradient may cause spatial trends which may affect the response under consideration. In such situations, the response may also depend on the spatial position of the experimental unit within a block. One way to overcome this situation is the application of suitable arrangement of treatments over plots within a block such that the arranged design is capable of completely eliminating the effects of defined components of a common trend. Such designs are known as *Trend Free Block* (TFB) designs (Bradley and Yeh 1980). These designs are constructed in such a manner that the treatment effects and the trend effects are orthogonal. (Bradley and Yeh 1980) introduced the concept of a TFB design along with the necessary and sufficient condition for the existence of such designs. A good number of work is available in literature which deals with different aspects of trend resistant block designs (Yeh and Bradley

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1983, Bhowmik 2013 etc). Bhowmik *et al*. (2012, 2014) obtained trend free designs in the presence of one directional and both directional neighbour effects from immediate neighbouring units. Trend free block designs in the presence of second order neighbour effects were also obtained by Bhowmik *et al*. (2015).

This article deals with Trend Resistant Neighbour Balanced Bipartite Block (TR-NBBPB) designs when there are two disjoint sets of treatments (one set may be tests and other may be controls. The interest here is to estimate the contrasts pertaining to test treatments (direct effects and neighbour effects) vs. control treatments with higher precision. Series of TR-NBBPB for comparing a treatment from set 1 to a treatment from set 2 have been developed.

2. MODEL AND DEFINITIONS

Let there are v (= v_1+v_2; v_1 treatments in set 1 and v_2 treatments in set 2) treatments and b blocks of size k each. We define the following model:

$$\mathbf{Y} = \mu\mathbf{1} + \Delta'_\tau\boldsymbol{\tau} + \Delta'_\delta\boldsymbol{\delta} + \Delta'_\gamma\gamma + \mathbf{D}'\boldsymbol{\beta} + \mathbf{Z}\boldsymbol{\rho} + \mathbf{e} \tag{2.1}$$

where, $\mathbf{Y}$ is a $n\times1$ vector of observations, μ is the general mean, $\mathbf{1}$ is the $n\times1$ vector of unity, Δ'_τ is $n\times v$ matrix of observations versus direct treatments of both the sets, $\boldsymbol{\tau}$ is $v\times1$ vector of direct treatment effects, Δ'_δ is $n\times v$ matrix of observations versus left neighbour treatments from both the sets, $\boldsymbol{\delta}$ is $v\times1$ vector of left neighbour effects, Δ'_γ is $n\times v$ matrix of observations versus right neighbour treatments, $\boldsymbol{\gamma}$ is $v\times1$ vector of right neighbour effects, $\mathbf{D}$' is $n\times b$ incidence matrix of observations versus blocks, $\boldsymbol{\beta}$ is $b\times1$ vector of block effects, $\boldsymbol{\rho}$ is $a\times1$ vector representing the trend effects. The matrix $\mathbf{Z}$, of order $n\times p$, is the matrix of coefficients which is given by $\mathbf{Z}=\mathbf{1}_b\otimes\mathbf{F}$ where $\mathbf{F}$ is $k\times a$ matrix with columns representing the (normalized) orthogonal polynomials and $\mathbf{e}$ is $n\times1$ vector of errors with E($\mathbf{e}$) = 0 and D($\mathbf{e}$) = $\sigma^2\mathbf{I}_n$.

Based on the above model, the following incidence matrices are defined:

$$\Delta_\tau\Delta'_\delta = \mathbf{M}_\delta = \begin{bmatrix} \mathbf{M}_{\delta1} & \mathbf{M}_{\delta2} \\ \mathbf{M}'_{\delta2} & \mathbf{M}_{\delta3} \end{bmatrix},$$

is a $(v_1+v_2)'(v_1+v_2)$ incidence matrix with $\mathbf{M}_{\delta1}$ as the incidence matrix of direct treatments of set 1 vs left neighbour treatments of set 1, $\mathbf{M}_{\delta2}$ as the incidence matrix of direct treatments of set 1 vs left neighbour treatments of set 2 and $\mathbf{M}_{\delta3}$ as incidence matrix of direct treatments of set 2 vs left neighbour treatments of set 2.

$$\Delta_\tau\Delta'_\gamma = \mathbf{M}_\gamma = \begin{bmatrix} \mathbf{M}_{\gamma1} & \mathbf{M}_{\gamma2} \\ \mathbf{M}'_{\gamma2} & \mathbf{M}_{\gamma3} \end{bmatrix},$$

where $\mathbf{M}_{\gamma1}$ is the incidence matrix of direct treatments of set 1 vs right neighbour treatments of set 1, $\mathbf{M}_{\gamma2}$ is the incidence matrix of direct treatments of set 1 vs right neighbour treatments of set 2 and $\mathbf{M}_{\gamma3}$ is the incidence matrix of direct treatments of set 2 vs right neighbour treatments of set 2.

$$\Delta_\delta\Delta'_\gamma = \mathbf{M} = \begin{bmatrix} \mathbf{M}_1 & \mathbf{M}_2 \\ \mathbf{M}'_2 & \mathbf{M}_3 \end{bmatrix},$$

where $\mathbf{M}_1$ is the incidence matrix of left neighbour treatments of set 1 vs right neighbour treatments of set 1, $\mathbf{M}_2$ is the incidence matrix of left neighbour treatments of set 1 vs right neighbour treatments of set 2 and $\mathbf{M}_3$ is the incidence matrix of left neighbour treatments of set 2 vs right neighbour treatments of set 2.

$$\Delta_\tau\mathbf{D}' = \mathbf{N}_\tau = \begin{bmatrix} \mathbf{N}_{\tau1} \\ \mathbf{N}_{\tau2} \end{bmatrix}$$

is $(v_1+v_2)\times b$ incidence matrix with $\mathbf{N}_{\tau1}$ as the incidence matrix of direct treatments of set 1 vs block and $\mathbf{N}_{\tau2}$ is the incidence matrix of direct treatments of set 2 vs block. Similarly,

$$\Delta_\delta\mathbf{D}' = \mathbf{N}_\delta = \begin{bmatrix} \mathbf{N}_{\delta1} \\ \mathbf{N}_{\delta2} \end{bmatrix} \text{ and } \Delta_\gamma\mathbf{D}' = \mathbf{N}_\gamma = \begin{bmatrix} \mathbf{N}_{\gamma1} \\ \mathbf{N}_{\gamma2} \end{bmatrix}.$$

Let

$\mathbf{r} = [\mathbf{r}'_{\tau1}\ \ \mathbf{r}'_{\tau2}]'$ is the $(v_1+v_2)\times1$ replication vector of direct treatments with $\mathbf{r}_{\tau1}$ as the replication vector of first set of treatments and $\mathbf{r}_{\tau2}$ as the replication of second set of treatments.

$\mathbf{r}_\delta = [\mathbf{r}'_{\delta1}\ \ \mathbf{r}'_{\delta2}]'$ is the $(v_1+v_2)\times1$ replication vector of the treatments appearing as left neighbour with $\mathbf{r}_{\delta1}$ as the replication vector of first set of treatments appearing as left neighbour and $\mathbf{r}_{\delta2}$ as the replication of second set of treatments appearing as left neighbour.

$\mathbf{r}_\gamma = [\mathbf{r}'_{\gamma1}\ \ \mathbf{r}'_{\gamma2}]'$ is the $(v_1+v_2)\times1$ replication vector of the treatments appearing as right neighbour with $\mathbf{r}_{\gamma1}$ as the replication vector of first set of treatments appearing as right neighbour and $\mathbf{r}_{\gamma 2}$ as the replication of second set of treatments appearing as right neighbour.

Further, $\mathbf{R}_{\tau} = \begin{bmatrix} \mathbf{R}_{\tau 1} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_{\tau 2} \end{bmatrix}$; $\mathbf{R}_{\delta} = \begin{bmatrix} \mathbf{R}_{\delta 1} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_{\delta 2} \end{bmatrix}$; $\mathbf{R}_{\gamma} = \begin{bmatrix} \mathbf{R}_{\gamma 1} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_{\gamma 2} \end{bmatrix}$.

Therefore, the information matrix for estimating the direct effect of treatments is obtained as

$$\mathbf{C}_{\tau} = \mathbf{C}_{11} - \mathbf{C}_{12}\mathbf{C}_{22}^{-}\mathbf{C}_{21} \tag{2.2}$$

with

$$\mathbf{C}_{11} = \begin{bmatrix} \mathbf{R}_{\tau 1} - \mathbf{N}_{\tau 1}\mathbf{K}^{-1}\mathbf{N}'_{\tau 1} - \frac{1}{b}\Delta_{\tau 1}\mathbf{Z}\mathbf{Z}'\Delta'_{\tau 1} & -\mathbf{N}_{\tau 1}\mathbf{K}^{-1}\mathbf{N}'_{\tau 2} - \frac{1}{b}\Delta_{\tau 1}\mathbf{Z}\mathbf{Z}'\Delta'_{\tau 2} \\ -\mathbf{N}_{\tau 2}\mathbf{K}^{-1}\mathbf{N}'_{\tau 1} - \frac{1}{b}\Delta_{\tau 2}\mathbf{Z}\mathbf{Z}'\Delta'_{\tau 1} & \mathbf{R}_{\tau 2} - \mathbf{N}_{\tau 2}\mathbf{K}^{-1}\mathbf{N}'_{\tau 2} - \frac{1}{b}\Delta_{\tau 2}\mathbf{Z}\mathbf{Z}'\Delta'_{\tau 2} \end{bmatrix}$$

$$_{12}\begin{bmatrix} \mathbf{M}_{\delta 1} - \mathbf{N}_{\tau 1}\mathbf{K}^{-1}\mathbf{N}'_{\delta 1} - \frac{1}{b}\boldsymbol{\Delta}_{\tau 1}\mathbf{Z}\mathbf{Z}'\boldsymbol{\Delta}'_{\delta 1} & \mathbf{M}_{\delta 2} - \mathbf{N}_{\tau 1}\mathbf{K}^{-1}\mathbf{N}'_{\delta 2} - \frac{1}{b}\boldsymbol{\Delta}_{\tau 1}\mathbf{Z}\mathbf{Z}'\boldsymbol{\Delta}'_{\delta 2} & \mathbf{M}_{\gamma 1} - \mathbf{N}_{\tau 1}\mathbf{K}^{-1}\mathbf{N}'_{\gamma 1} - \frac{1}{b}\boldsymbol{\Delta}_{\tau 1}\mathbf{Z}\mathbf{Z}'\boldsymbol{\Delta}'_{\gamma 1} & \mathbf{M}_{\gamma 2} - \mathbf{N}_{\tau 1}\mathbf{K}^{-1}\mathbf{N}'_{\gamma 2} - \frac{1}{b}\boldsymbol{\Delta}_{\tau 1}\mathbf{Z}\mathbf{Z}'\boldsymbol{\Delta}'_{\gamma 2} \\ \mathbf{M}'_{\delta 2} - \mathbf{N}_{\tau 2}\mathbf{K}^{-1}\mathbf{N}'_{\delta 1} - \frac{1}{b}\boldsymbol{\Delta}_{\tau 2}\mathbf{Z}\mathbf{Z}'\boldsymbol{\Delta}'_{\delta 1} & \mathbf{M}_{\delta 3} - \mathbf{N}_{\tau 2}\mathbf{K}^{-1}\mathbf{N}'_{\delta 2} - \frac{1}{b}\boldsymbol{\Delta}_{\tau 2}\mathbf{Z}\mathbf{Z}'\boldsymbol{\Delta}'_{\delta 2} & \mathbf{M}'_{\gamma 2} - \mathbf{N}_{\tau 2}\mathbf{K}\ \ \mathbf{N}'_{\gamma 1} - -\boldsymbol{\Delta}_{\tau 2}\mathbf{Z}\mathbf{Z}'\boldsymbol{\Delta}'_{\gamma 1} & \mathbf{M}_{\gamma 3} - \mathbf{N}_{\tau 2}\mathbf{K}\ \ \mathbf{N}'_{\gamma 2} - -\boldsymbol{\Delta}_{\tau 2}\mathbf{Z}\mathbf{Z}'\boldsymbol{\Delta}'_{\gamma 2} \end{bmatrix}$$

and

$$\mathbf{C}_{22} = \begin{bmatrix} \mathbf{R}_{\delta 1} - \mathbf{N}_{\delta 1}\mathbf{K}^{-1}\mathbf{N}'_{\delta 1} - \frac{1}{b}\boldsymbol{\Delta}_{\delta 1}\mathbf{Z}\mathbf{Z}'\boldsymbol{\Delta}'_{\delta 1} & -\mathbf{N}_{\delta 1}\mathbf{K}^{-1}\mathbf{N}'_{\delta 2} - \frac{1}{b}\boldsymbol{\Delta}_{\delta 1}\mathbf{Z}\mathbf{Z}'\boldsymbol{\Delta}'_{\delta 2} & \mathbf{M}_{1} - \mathbf{N}_{\delta 1}\mathbf{K}^{-1}\mathbf{N}'_{\gamma 1} - \frac{1}{b}\boldsymbol{\Delta}_{\delta 1}\mathbf{Z}\mathbf{Z}'\boldsymbol{\Delta}'_{\gamma 1} & \mathbf{M}_{2} - \mathbf{N}_{\delta 1}\mathbf{K}^{-1}\mathbf{N}'_{\gamma 2} - \frac{1}{b}\boldsymbol{\Delta}_{\delta 1}\mathbf{Z}\mathbf{Z}'\boldsymbol{\Delta}'_{\gamma 2} \\ -\mathbf{N}_{\delta 2}\mathbf{K}^{-1}\mathbf{N}'_{\delta 1} - \frac{1}{b}\boldsymbol{\Delta}_{\delta 2}\mathbf{Z}\mathbf{Z}'\boldsymbol{\Delta}'_{\delta 1} & \mathbf{R}_{\delta 2} - \mathbf{N}_{\delta 2}\mathbf{K}^{-1}\mathbf{N}'_{\delta 2} - \frac{1}{b}\boldsymbol{\Delta}_{\delta 2}\mathbf{Z}\mathbf{Z}'\boldsymbol{\Delta}'_{\delta 2} & \mathbf{M}'_{2} - \mathbf{N}_{\delta 2}\mathbf{K}^{-1}\mathbf{N}'_{\gamma 1} - \frac{1}{b}\boldsymbol{\Delta}_{\delta 2}\mathbf{Z}\mathbf{Z}'\boldsymbol{\Delta}'_{\gamma 1} & \mathbf{M}_{3} - \mathbf{N}_{\delta 2}\mathbf{K}^{-1}\mathbf{N}'_{\gamma 2} - \frac{1}{b}\boldsymbol{\Delta}_{\delta 2}\mathbf{Z}\mathbf{Z}'\boldsymbol{\Delta}'_{\gamma 2} \\ \mathbf{M}'_{1} - \mathbf{N}_{\gamma 1}\mathbf{K}^{-1}\mathbf{N}'_{\delta 1} - \frac{1}{b}\boldsymbol{\Delta}_{\gamma 1}\mathbf{Z}\mathbf{Z}'\boldsymbol{\Delta}'_{\delta 1} & \mathbf{M}'_{2} - \mathbf{N}_{\gamma 1}\mathbf{K}^{-1}\mathbf{N}'_{\delta 2} - \frac{1}{b}\boldsymbol{\Delta}_{\gamma 1}\mathbf{Z}\mathbf{Z}'\boldsymbol{\Delta}'_{\delta 2} & \mathbf{R}_{\gamma 1} - \mathbf{N}_{\gamma 1}\mathbf{K}^{-1}\mathbf{N}'_{\gamma 1} - \frac{1}{b}\boldsymbol{\Delta}_{\gamma 1}\mathbf{Z}\mathbf{Z}'\boldsymbol{\Delta}'_{\gamma 1} & -\mathbf{N}_{\gamma 1}\mathbf{K}^{-1}\mathbf{N}'_{\gamma 2} - \frac{1}{b}\boldsymbol{\Delta}_{\gamma 1}\mathbf{Z}\mathbf{Z}'\boldsymbol{\Delta}'_{\gamma 2} \\ \mathbf{M}_{2} - \mathbf{N}_{\gamma 2}\mathbf{K}^{-1}\mathbf{N}'_{\delta 1} - \frac{1}{b}\boldsymbol{\Delta}_{\gamma 2}\mathbf{Z}\mathbf{Z}'\boldsymbol{\Delta}'_{\delta 1} & \mathbf{M}'_{3} - \mathbf{N}_{\gamma 2}\mathbf{K}^{-1}\mathbf{N}'_{\delta 2} - \frac{1}{b}\boldsymbol{\Delta}_{\gamma 2}\mathbf{Z}\mathbf{Z}'\boldsymbol{\Delta}'_{\delta 2} & -\mathbf{N}_{\gamma 2}\mathbf{K}^{-1}\mathbf{N}'_{\gamma 1} - \frac{1}{b}\boldsymbol{\Delta}_{\gamma 2}\mathbf{Z}\mathbf{Z}'\boldsymbol{\Delta}'_{\gamma 1} & \mathbf{R}_{\gamma 2} - \mathbf{N}_{\gamma 2}\mathbf{K}^{-1}\mathbf{N}'_{\gamma 2} - \frac{1}{b}\boldsymbol{\Delta}_{\gamma_2}\mathbf{Z}\mathbf{Z}'\boldsymbol{\Delta}'_{\gamma 2} \end{bmatrix}$$

Following are some definitions useful in the context of the present paper:

Definition 2.1: A block design with neighbour effects for two disjoint sets of treatments is said to be Neighbour Balanced Bipartite Block (NBBPB) design if every treatment from first set has every other treatment from same set appearing constant (say, μ^*_{11}) number of times as a right and as a left neighbour, every treatment from first set has every treatment from second set appearing constant (say, μ^*_{12}) number of times as a right and as a left neighbour and every treatment from second set has every other treatment from same set appearing constant (say, μ^*_{22}) number of times as a right and as a left neighbour.

Definition 2.2: A NBBPB design is called a Trend Resistant NBBPB (TR-NBBPB) design if the adjusted treatment sum of squares arising from direct effects of treatments and neighbour effects of treatments under model 2.1 is same as the adjusted treatment sum of squares under the usual block model with neighbour effects without trend component.

Result 2.1: A NBBPB design incorporating trend component is said to be completely trend resistant or trend free iff

$$\boldsymbol{\Delta}_{\tau}\mathbf{Z} = 0, \boldsymbol{\Delta}_{\tau 1}\mathbf{Z} = 0, \boldsymbol{\Delta}_{\tau 2}\mathbf{Z} = 0,$$
$$\boldsymbol{\Delta}_{\delta}\mathbf{Z} = 0, \boldsymbol{\Delta}_{\delta 1}\mathbf{Z} = 0, \boldsymbol{\Delta}_{\delta 2}\mathbf{Z} = 0,$$
$$\boldsymbol{\Delta}_{\gamma}\mathbf{Z} = 0, \boldsymbol{\Delta}_{\gamma 1}\mathbf{Z} = 0 \text{ and } \boldsymbol{\Delta}_{\gamma 2}\mathbf{Z} = 0$$

3. METHODS OF CONSTRUCTING TR-NBBPB DESIGNS

Following are some methods for constructing TR-NBBPB designs. In all the cases, it is assumed that the blocks are circular i.e. A neighbour balanced block (NBB) with two-sided neighbour effects is said to be circular if the treatment in the left border is the same as the treatment in the right-end inner plot and the treatment in the right border is the same as the treatment in the left-end inner plot.

Method 3.1: For $v = sm + 1$, (v being prime or prime power and > 3) develop s initial block(s) as $x^w, x^{w+s}, x^{w+2s}, \ldots x^{w+(m-1)s}$ modulo v for all $w = 0,1, \ldots, s$-1 (Tomar *et al.* 2005). Here, x is the primitive element of GF (v). Develop the s initial block(s) cyclically modulo v. Now, out of v consider p groups of u treatments each such that $pu< v$-3. Replace the treatments in group 1 by any one treatment not in the group. Repeat this for all the groups. For convenience the groups are formed by taking treatments in reverse order. The resulting design will be an incomplete TR-NBBPB design with parameters $v_1 = v\text{-}p(u+1)$, $v_2 = p$, $b = sv$, $r_1 = sm$, $r_2 = usm$, $k = m$, $\mu^*_{11} = 1$, $\mu^*_{12} = \mu$, $\mu^*_{22} = \mu^2$ and number of times each treatment appears in all the positions is s.

The information matrix for estimating the contrast pertaining to the direct effects of treatments for the above TR-NBBPB designis obtained as

$$\mathbf{C}_{\tau} = \frac{(k-3)}{(k-2)}\begin{bmatrix} v\mathbf{I}_{v_1} - \mathbf{1}_{v_1}\mathbf{1}'_{v_1} & -u\mathbf{1}_{v_1}\mathbf{1}'_{v_2} \\ -u\mathbf{1}_{v_2}\mathbf{1}'_{v_1} & uv\mathbf{I}_{v_2} - u^2\mathbf{1}_{v_2}\mathbf{1}'_{v_2} \end{bmatrix} \quad (3.1)$$

The variance of any estimated elementary contrast among the direct effects (left neighbour effects, right neighbour effects) pertaining to treatments of set 1 is

$$\mathrm{V}_{1\tau} = \mathrm{V}_{1\delta} = \mathrm{V}_{1\gamma} = \frac{2(k-2)}{v(k-3)}\sigma^2$$

and the variance of any estimated elementary contrast among the direct effects (left neighbour effects, right neighbour effects) pertaining to treatments of set 1 vs. set 2 is

$$\mathrm{V}_{12\tau} = \mathrm{V}_{12\delta} = \mathrm{V}_{12\gamma} = \frac{(k-2)(u+1)}{uv(k-3)}\sigma^2$$

Remark 3.1: It can be noted that the information matrices for estimating the contrast pertaining to left and right neighbour effects are exactly same as Equation (3.1).

Example 3.1: Let $m = 5$ and $s = 2$, then the following two initial blocks for $w = 0$ and $w = 1$ can be obtained modulo 11:

1 4 5 9 3 and 2 8 10 7 6

Developing these blocks cyclically following design is obtained which is neighbour balanced:

1	4	5	9	3
2	5	6	10	4
3	6	7	11	5
4	7	8	1	6
5	8	9	2	7
6	9	10	3	8
7	10	11	4	9
8	11	1	5	10
9	1	2	6	11
10	2	3	7	1
11	3	4	8	2
2	8	10	7	6
3	9	11	8	7
4	10	1	9	8
5	11	2	10	9
6	1	3	11	10
7	2	4	1	11
8	3	5	2	1
9	4	6	3	2
10	5	7	4	3
11	6	8	5	4
1	7	9	6	5

Let $p = 2$ groups and $u = 2$ treatments, then $pu = 4$. Replace last 4 treatments by 2 treatments as follows:

Group 1: treatments (11, 10) by 7 and
Group 2: treatments (9, 8) by 6

Following TR-NBBPB design is obtained with parameters $v_1 = 5$, $v_2 = 2$, $b = 22$, $r_1 = 10$, $r_2 = 20$, $k = 5$, $\mu^*_{11} = 1$, $\mu^*_{12} = 2$, $\mu^*_{22} = 4$ and each treatment appears twice in every position:

-2	-1	0	1	2
1	4	5	6	3
2	5	6	7	4
3	6	7	7	5
4	7	6	1	6
5	6	6	2	7
6	6	7	3	6
7	7	7	4	6
6	7	1	5	7
6	1	2	6	7
7	2	3	7	1
7	3	4	6	2
2	6	7	7	6
3	6	7	6	7
4	7	1	6	6
5	7	2	7	6
6	1	3	7	7
7	2	4	1	7
6	3	5	2	1
6	4	6	3	2
7	5	7	4	3
7	6	6	5	4
1	7	6	6	5

Orthogonal trend component of degree one without normalization (Fisher and Yates 1963) is given in the upper row and

$$\mathbf{F} = \begin{bmatrix} \frac{-2}{\sqrt{10}} & \frac{-1}{\sqrt{10}} & 0 & \frac{1}{\sqrt{10}} & \frac{2}{\sqrt{10}} \end{bmatrix}'$$

$$= \begin{bmatrix} -0.63 & -0.31 & 0 & 0.31 & 0.63 \end{bmatrix}'$$

The Information matrices for estimating the contrast pertaining to direct, left neighbour and right neighbour effects are given as

$$\mathbf{C}_{\hat{o}} = \mathbf{C}_{\ddot{a}} = \mathbf{C}_{\gamma} = \frac{2}{3}\begin{bmatrix} 11\mathbf{I}_5 - \mathbf{1}_5\mathbf{1}_5' & -2\mathbf{1}_5\mathbf{1}_3' \\ -2\mathbf{1}_3\mathbf{1}_5' & 22\mathbf{I}_3 - 4\mathbf{1}_3\mathbf{1}_3' \end{bmatrix}$$

with variances

$$\mathrm{V}_{1\tau} = \mathrm{V}_{1\delta} = \mathrm{V}_{1\gamma} = 0.2727\sigma^2 \text{ and}$$

$$\mathrm{V}_{12\tau} = \mathrm{V}_{12\delta} = \mathrm{V}_{12\gamma} = 0.2045\sigma^2$$

Hence the design so obtained is more efficient for estimating contrasts pertaining to treatments of set 1 vs set 2.

Method 3.2: Consider a circular complete NBB design obtained by taking the j^{th} block (j = 1,2,…, v-1, v being prime ≥ 5) of the design as v, j, $2j$,…,$(v-1)j$ modulo v (Azais *et al*. 1993). Out of v consider p groups of u treatments each such that $pu<v$-3. Replace the treatments in group 1 by any one treatment not in the group. Repeat this for all the groups. Fold-over the blocks of the design obtained i.e. appending the vertical mirror image of the blocks below the original set of blocks. The resulting design will be a TR-NBBPB design with parameters $v_1 = v - p(u+1)$, $v_2 = p$, $b = 2(v-1)$, $r_1 = 2(v-1)$, $r_2 = 2u(v-1)$, $k = v$, $\mu^*_{11} = 2$, $\mu^*_{12} = 2\mu$ and $\mu^*_{22} = 2\mu^2$.

The information matrices for estimating the contrast pertaining to direct, left neighbour and right neighbour effects for the above TR-NBBPB designs is given as

$$\mathbf{C}_{\hat{o}} = \mathbf{C}_{\ddot{a}} = \mathbf{C}_{\gamma}$$

$$= \frac{2(v-3)}{(v-2)}\begin{bmatrix} v\mathbf{I}_{v_1} - \mathbf{1}_{v_1}\mathbf{1}_{v_1}' & -u\mathbf{1}_{v_1}\mathbf{1}_{v_2}' \\ -u\mathbf{1}_{v_2}\mathbf{1}_{v_1}' & uv\mathbf{I}_{v_2} - u^2\mathbf{1}_{v_2}\mathbf{1}_{v_2}' \end{bmatrix} \qquad (3.2)$$

with

$$\mathrm{V}_{1\tau} = \mathrm{V}_{1\delta} = \mathrm{V}_{1\gamma} = \frac{(v-2)}{v(v-3)}\sigma^2$$

$$\mathrm{V}_{12\tau} = \mathrm{V}_{12\delta} = \mathrm{V}_{12\gamma} = \frac{(v-2)(u+1)}{2uv(v-3)}\sigma^2$$

Example 3.2: Consider a circular NBB design for v=7 treatments replicated 6 times each in 6 complete blocks. Following the procedure described above with p=2 and u=1, we substitute treatment number 7 by 5 and 6 by 4 and juxtapose the fold-over design. A TR-NBBPB design is obtained with parameters v_1 = 3, v_2 = 2, b = 12, r_1 = 12, r_2 = 24, k = 7, μ^*_{11} = 2, μ^*_{12} = 4 and μ^*_{22} = 8.

	1	2	3	4	5	6	7	1
6	1	3	5	7	2	4	6	1
5	1	4	7	3	6	2	5	1
4	1	5	2	6	3	7	4	1
3	1	6	4	2	7	5	3	1
2	1	7	6	5	4	3	2	1

Replacing 7 by 5 and 6 by 4

	-3	-2	-1	0	1	2	3	
5	1	2	3	4	4	5	5	1
4	1	3	4	5	2	4	5	1
5	1	4	5	3	5	2	4	1
4	1	4	2	5	3	5	4	1
3	1	5	4	2	5	4	3	1
2	1	5	5	4	4	3	2	1
1	5	5	4	4	3	2	1	5
1	5	4	2	5	4	3	1	4
1	4	2	5	3	5	4	1	5
1	4	5	3	5	2	4	1	4
1	3	4	5	2	4	5	1	3
1	2	3	4	4	5	5	1	2

Fold-over

The information matrices for estimating direct, left neighbour and right neighbour effects is given as

$$\mathbf{C}_{\tau} = \mathbf{C}_{\delta} = \mathbf{C}_{\gamma} = \frac{1}{5}\begin{bmatrix} 56\mathbf{I}_3 - 8\mathbf{1}_3\mathbf{1}_3' & -16\mathbf{1}_3\mathbf{1}_2' \\ -16\mathbf{1}_2\mathbf{1}_3' & 112\mathbf{I}_2 - 32\mathbf{1}_2\mathbf{1}_2' \end{bmatrix}$$

with variances

$$\mathrm{V}_{1\tau} = \mathrm{V}_{1\delta} = \mathrm{V}_{1\gamma} = 0.1786\sigma^2 \text{ and}$$

$$\mathrm{V}_{12\tau} = \mathrm{V}_{12\delta} = \mathrm{V}_{12\gamma} = 0.1339\sigma^2$$

Remark 3.2.1: In previous method, one can obtain a NBB design by writing the treatments in systematic order within a block with a difference of 1, 2,…,v-1 between the treatments (modulo v) in the consecutive blocks. Taking these (v-1) blocks as initial blocks and developing $v(v-1)$ blocks by clock wise rotating the treatments we can generate another plan. Using the substitution method to the above plan we get a TR-NBBPB design with parameters $v_1 = v - p(u+1)$, $v_2 = p$, $b = v(v-1)$, $r_1 = v(v-1)$, $r_2 = uv(v-1)$, $k = v$, $\mu^*_{11} = v$, $\mu^*_{12} = uv$ and $\mu^*_{22} = vu^2$. For this design the information matrices and variances are given as

$$\mathbf{C}_\tau = \mathbf{C}_\delta = \mathbf{C}_\gamma = \frac{v(v-3)}{(v-2)}\begin{bmatrix} v\mathbf{I}_{v_1} - \mathbf{1}_{v_1}\mathbf{1}'_{v_1} & -u\mathbf{1}_{v_1}\mathbf{1}'_{v_2} \\ -u\mathbf{1}_{v_2}\mathbf{1}'_{v_1} & uv\mathbf{I}_{v_2} - u^2\mathbf{1}_{v_2}\mathbf{1}'_{v_2} \end{bmatrix}$$

with

$$\mathrm{V}_{1\tau} = \mathrm{V}_{1\delta} = \mathrm{V}_{1\gamma} = \frac{(v-2)}{(v-3)}\sigma^2 \text{ and}$$

$$\mathrm{V}_{12\tau} = \mathrm{V}_{12\delta} = \mathrm{V}_{12\gamma} = \frac{(v-2)(u+1)}{uv^2(v-3)}\sigma^2$$

Method 3.3: Consider a Balanced Incomplete Block (BIB) design with parameters v^*, b^*, r^*, k^* and λ^*. Augment v_2 treatments to each block of this design such that $k^* + v_2$ is a prime number. Obtain $k^* + v_2$-1 blocks from each block by arranging the treatments in systematic order within a block with a difference of 1,2,…,v-1 between the treatments (modulo v) in the consecutive blocks. Juxtaposing all the blocks of design and all the blocks of its fold-over form we get the TR-NBBPB design with parameters $v_1 = v^*$, v_2, $b = 2b^*(k^*+v_2-1)$, $r_1 = 2r^*(k^*+v_2-1)$, $r_2 = 2b^*(k^*+v_2-1)$, $k = k^*+v_2$, $\mu^*_{11} = 2\lambda^*$, $\mu^*_{12} = 2r^*$ and $\mu^*_{22} = 2b^*$.

The information matrices for estimating direct, left neighbour and right neighbour effects is given as

$$\mathbf{C}_\tau = \mathbf{C}_\delta = \mathbf{C}_\gamma = \frac{(k-1)}{(k-2)}$$
$$\begin{bmatrix} \left(k^2 - 2k - 2\lambda^*\right)\mathbf{I}_{v_1} - \mathbf{1}_{v_1}\mathbf{1}'_{v_1} & -(k-2)\mathbf{1}_{v_1}\mathbf{1}'_{v_2} \\ -(k-2)\mathbf{1}_{v_2}\mathbf{1}'_{v_1} & b^*k\mathbf{I}_{v_2} - b^*\mathbf{1}_{v_2}\mathbf{1}'_{v_2} \end{bmatrix}$$

Example 3.3: Consider a BIB design with parameters $v^* = 4$, $b^* = 6$, $r^* = 3$, $k^* = 2$ and $\lambda^* = 1$. Augmenting treatment 5, 6 and 7 in each block of the design, rearranging, juxtaposing its fold-over, the following TR-NBBPB design with parameters $v_1 = 4$, $v_2 = 3$, $b = 46$, $r_1 = 24$, $r_2 = 48$, $k = 5$, $\mu^*_{11} = 2$, $\mu^*_{12} = 6$ and $\mu^*_{22} = 12$ is obtained:

	-2	-1	0	1	2	
7	1	2	5	6	7	1
6	1	5	7	2	6	1
5	1	6	2	7	5	1
2	1	7	6	5	2	1
7	1	3	5	6	7	1
6	1	5	7	3	6	1
5	1	6	3	7	5	1
3	1	7	6	5	3	1
7	1	4	5	6	7	1
6	1	5	7	4	6	1
5	1	6	4	7	5	1
4	1	7	6	5	4	1
7	2	3	5	6	7	2
6	2	5	7	3	6	2
5	2	6	3	7	5	2
3	2	7	6	5	3	2
7	2	4	5	6	7	2
6	2	5	7	4	6	2
5	2	6	4	7	5	2
4	2	7	6	5	4	2
7	3	4	5	6	7	3
6	3	5	7	4	6	3
5	3	6	4	7	5	3
4	3	7	6	5	4	3
1	7	6	5	2	1	7
1	6	2	7	5	1	6
1	5	7	2	6	1	5
1	2	5	6	7	1	2
1	7	6	5	3	1	7
1	6	3	7	5	1	6
1	5	7	3	6	1	5
1	3	5	6	7	1	3
1	7	6	5	4	1	7
1	6	4	7	5	1	6
1	5	7	4	6	1	5
1	4	5	6	7	1	4
2	7	6	5	3	2	7
2	6	3	7	5	2	6
2	5	7	3	6	2	5
2	3	5	6	7	2	3
2	7	6	5	4	2	7
2	6	4	7	5	2	6
2	5	7	4	6	2	5
2	4	5	6	7	2	4
3	7	6	5	4	3	7
3	6	4	7	5	3	6
3	5	7	4	6	3	5
3	4	5	6	7	3	4

For this design, the information matrices and variances are obtained as

$$\mathbf{C}_\tau = \mathbf{C}_\delta = \mathbf{C}_\gamma = \frac{4}{3}\begin{bmatrix} 13\mathbf{I}_4 - \mathbf{1}_4\mathbf{1}_4' & -3\mathbf{1}_4\mathbf{1}_3' \\ -3\mathbf{1}_3\mathbf{1}_4' & 30\mathbf{I}_3 - 6\mathbf{1}_3\mathbf{1}_3' \end{bmatrix},$$

$$\mathrm{V}_{1\tau} = \mathrm{V}_{1\delta} = \mathrm{V}_{1\gamma} = 0.1154\sigma^2 \text{ and}$$

$$\mathrm{V}_{12\tau} = \mathrm{V}_{12\delta} = \mathrm{V}_{12\gamma} = 0.0808\sigma^2$$

It is seen that all the TR-NBBPB designs obtained are totally balanced for estimating direct and neighbour effects of treatments and are capable of completely eliminating the effects of a common trend. Since the designs are trend-resistant, the analysis may be carried out as per the usual procedure of a NBB design that consist of direct effects and neighbour effects besides the block effects. For making comparisons between treatments from two sets, appropriate contrasts need to be defined. The standard statistical software can be used for this purpose.

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